Approximate shortest paths on weighted disks

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In this paper, we study optimal obstacle-avoiding paths from a source point s to a target point t in the 2-dimensional plane. A general version of the shortest path problem allows the two-dimensional space to be subdivided into regions. Each of the regions has a (non-negative) weight associated to it, representing the cost per unit distance of traveling in that region. This variant, called the Weighted Region Problem (WRP), was proposed by Mitchell and Papadimitriou [3]. They propose an approximating algorithm which computes a $(1+\varepsilon)$-approximation path in $O(n^8 \log \frac{nNW}{w\varepsilon})$ time, where N is the maximum integer coordinate of any vertex of the subdivision, W (respectively, w) is the maximum finite (respectively, minimum non-zero) integer weight assigned to faces of the subdivision.

Recently, it has been shown that the WRP cannot be solved exactly within the Algebraic Computation Model over the Rational Numbers (ACM$\mathbb{Q}$) [2], i.e., the solutions to some instances of the WRP cannot be expressed as a closed formula in ACM$\mathbb{Q}$.

This result probably explains the lack of exact algorithms for the WRP, and the fact that several authors propose algorithms for computing approximated paths. The most common scheme followed in the literature is to position Steiner points, and then build a graph by connecting pairs of Steiner points, see, e.g., [1, 4]. An approximate solution is constructed by finding a shortest path in this graph, by using well-known combinatorial algorithms (e.g., Dijkstra's algorithm).

Let D be a set of disjoint disks in the plane, and s, t be two points. To compute a shortest path, which avoids D, between s and t, we can use an algorithm based on Dijkstra's shortest path algorithm. However, we are not aware of any work where the shortest path is allowed to go through the disks $D_i \in D$, where each D_i has a non-negative value k_i associated to it.

It is straightforward to prove that if the weight of the disks is at least $\frac{\pi}{2}$, then the disks act as obstacles, and the problem can be solved in $O(|D|^2 \log |D|)$ time. Thus, we focus in finding an algorithm to compute an approximate shortest path from s to t, when $0 < k_i < \frac{\pi}{2}$.

Our first approach is the particular case in which $|D| = 1$, and s is a fixed source point on the boundary of the disk. In this case we obtained a promising result: a $(1+\varepsilon)$-approximation for the shortest path by carefully placing Steiner points on the boundary of the disk. In addition, we are currently working on the generalization of this partial result to the case of multiple disks.

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