Algorithms for efficient solar tracking in CSP plants

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Abstract

Increasing the efficiency in green energy production is mandatory to reduce dependency on fossil fuels. Capturing and storing solar energy is an appealing alternative, but optimizing energy collection with no damage to components of solar plants is a complex problem. In this work, some geometric optimization problems for solar tracking in Concentrated Solar Power plants based on Parabolic Through Collectors are addressed. Using nice properties of a solution, we propose efficient algorithms for optimal scheduling in solar tracking tasks that can be adapted for other solar plants.

1 Introduction

Concentrated Solar Power (CSP) plants are an effective alternative to photovoltaic technologies, as it has the capacity of storing the energy captured from the sun. Parabolic Trough Collectors (PTC) systems are one of the most widespread CSP plants around the globe, including more than 40 plants in Spain alone. PTC systems are composed of a parabolic-shaped surface reflecting the sun rays to a Heat Collector Element (HCE) located at the focus of the parabola. The parabolic-shaped mirror surface together with three HCEs forms a Solar Collector Element (SCE), and 4 SCEs are a Solar Collector Assembly (SCA). For a full decomposition of elements in the solar field of PTC plants, the reader is referred to [1].

During normal operation of PTC plants, SCAs are instructed to follow the sun so that the maximum energy can be collected, see Figure 1. Providing tracking systems to simultaneously improve accuracy and reduce operational cost is a seminal research area in solar plants. Methods for optimizing trackers in plants with arbitrary design and geometry have been proposed in the area of renewable energy [4, 2, 3]. When the operating conditions are optimal, a perfect tracking of the sun results in maximal energy collection.

Considering this scenario, the ray incidence over the HCE for different SCA and solar angles is expected to be unimodal. However, the shape of the function can change due to several errors, such as installation errors of some components of the SCA, cracks/dirt in the mirror surface, HCE bending and vertical/horizontal displacements due to mechanical stress, among others. This work raises some geometric problems to optimize the tracking system considering any shape for the ray incidence function. To the best of our knowledge, we are the first considering the optimization of the solar tracking while reducing the movements of the SCA in a PTC plant.

The rest of the paper is organized as follows: Section 2 provides the necessary background and the definition of the optimization problems; the algorithms are outlined in Sections 3 and 4.

2 Preliminaries

For this initial study, we assume that the weather conditions are constant throughout the day. Thus, *solar irradiance* over the HCE can be expressed as a function $z = f(x, y)$, where the (x, y) coordinates represent the Solar Collector Assembly and the sun angular displacements, respectively, while z corresponds to the number of rays touching the HCE. Since there is no change in the initial conditions, the 3D surface corresponding to f can be interpreted as a shifted 2D curve as illustrated in Figure 2. This visualization allow us to redefine the function as $z = f(\theta)$, where θ represents the difference between the sun and the SCA angular position. Solar tracking is discrete in PTC plants; hence, f can be defined as a step function with n steps as follows:

$$f(\theta) = \sum_{i=1}^{n} \alpha_i \delta_{S_i}(\theta), \tag{1}$$

where α_i is the number of rays touching the HCE in the step $S_i \in f$, and δ_{S_i} is a binary function indicating if $\theta \in S_i$.

Using a ray-tracing software, f can be obtained by moving the sun in a fixed axis with the SCA at 90º. The events at which the sun rays start/end intersecting the HCE can occur at any angular difference between the sun and the SCA position; hence, the length of the steps in f can be a real number. However, in this

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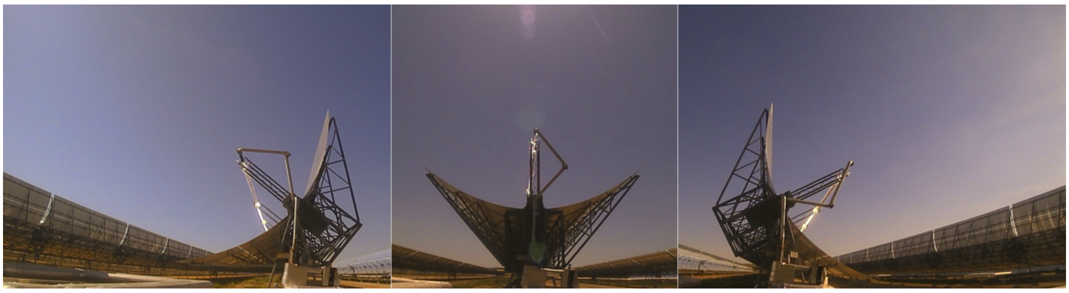

Figure 1: Solar tracking example in a real PTC plant.

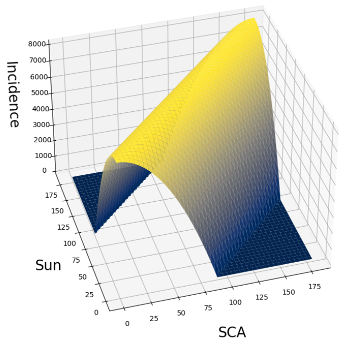

Figure 2: Ray incidence over the HCE depending on the angular position of the sun and the SCA for a simple case, assuming that the SCE has a perfect parabolic shape.

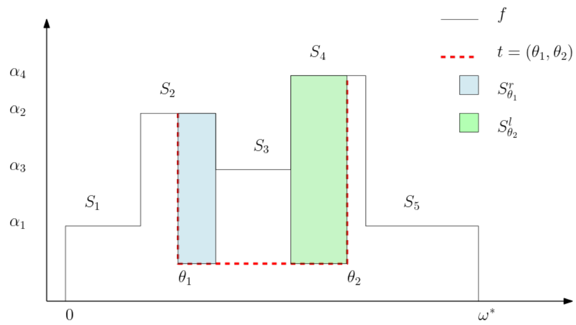

Figure 3: Main elements defining the ray incidence function f. t is a tracking interval from θ_1 to θ_2. Total irradiance in t (g_t) is the area within t below the curve. We consider f shifted to the range $(0, \omega^*)$.

abstract we consider the case in which these numbers are approximated as rational numbers (e.g., accurate to within one thousandth of a unit). This is standard in real-world applications, and involves the computational representation of real numbers. In the rational case, the problem can be reduced to one in which the solar irradiance function has steps with integer length.

2.1 The problems

Let f:$\mathbb{R} \to \mathbb{N}$ be a step function with n steps, defined as in equation 1. Let $S = \{S_1, \cdots, S_n\}$ be the set of steps of f. The elements of S are disjoint, ordered by x, and there is no gap between consecutive elements. A step $S_i \in S$ is an interval of the form $[\theta_{i_1}, \theta_{i_2})$, where $\theta_{i_1}, \theta_{i_2}$ are the edges of the step; let E be the set containing all the edges of S. The ray incidence over a step S_i is defined by α_{S_i}, its length as $l_{S_i} \in \mathbb{N}$, and the associated gain as $g_{S_i} = \alpha_{S_i} l_{S_i}$. For convenience, if $\theta \in S_i$, then $\theta \in S$, $\alpha_\theta = \alpha_{S_i}$, and S^l_θ (S^r_θ) is the portion of S_i from its left (right) edge to θ. In addition, for a tracking interval $t = (\theta_i, \theta_j)$ s.t. $\theta_i \in S_i$, $\theta_j \in S_j$, and $i \leq j$, the length of t is $l_t = \theta_j - \theta_i$, and its total gain can be defined by:

$$g_t = \begin{cases} g_{S^r_{\theta_i}} + g_{S^l_{\theta_j}} + \sum_{k=i+1}^{j-1} g_{S_k} & j > i \\ g_{S^r_{\theta_i}} - g_{S^r_{\theta_j}} & i = j \end{cases} \quad (2)$$

For a multiset $T = \{t_1, \ldots, t_k\}$ the total solar irradiance (gain) of the set is $I_T = \sum g_{t_i}$, and the total length is defined as $L_T = \sum l_{t_i}$. Finally, ω^* is the total length of S, and the initial position of the SCA w.r.t the sun is θ_0. See Figure 3 for an overview of the described notations.

We formulate two optimization problems of particular interest for solar tracking in CSP plants. The first problem looks for the minimum number of movements of the SCA such that the solar irradiance intersecting the HCE at any moment is preserved within a given range. The second one addresses to optimize the total solar irradiance intersecting the HCE with a limited number of allowed movements. More formally:

Problem 1 *(Min-Tracking, or MT-Problem): Given a step function f defined on $[0, \omega^*]$, and two real numbers u_1, u_2, find a set of intervals $T^* = \{t_1, \ldots, t_m\}$ of minimum size s.t. $t_i \subseteq [0, \omega^*]$, $\forall \theta \in t_i$, $u_1 \leq \alpha_\theta \leq u_2$ and $L_{T^*} + \theta_0 = \omega^*$.*

Problem 2 *(Maximal Energy Collection, or MEC-Problem): Given a step function f defined on $[0, \omega^*]$ and $m \in \mathbb{N}$, find a set of intervals $T^* = \{t_1, \ldots, t_j\}$ s.t. $t_i \subseteq [0, \omega^*]$, $|T^*| \leq m$, $L_{T^*} \leq \omega^*$, and I_{T^*} is maximal.*

3 Minimum Tracking

The analysis of Problem 1 must consider the initial position of the SCA with respect to the sun position. Two cases are possible: the SCA is in a feasible configuration, i.e. $u_1 \leq \alpha_{\theta_0} \leq u_2$; or the SCA is violating this restriction. In the former, it can be readily noticed that the optimal solution is to wait while the sun moves until a non-feasible state is reached. Therefore, we can assume, without lost of generality, that the SCA starts from a non-feasible configuration.

Theorem 1 *Let $t^* = (\theta_i, \theta_j)$ be a maximum tracking interval in f such that for any $\theta \in t^*$, $u_1 \leq \alpha_\theta \leq u_2$. If the SCA initially violates the boundary conditions, then the minimum possible cardinality of a solution that satisfies conditions of Problem 1 is $\lceil \frac{\omega^* - \theta_0}{l_{t^*}} \rceil$.*

Proof. Since the SCA violates the boundary condition, it needs to be moved to a feasible configuration. Let us assume that such feasible configuration initiates at θ_i and when the sun reaches θ_j the SCA moves again to θ_i. In such case it is clear that the SCA has rotated $\lceil \frac{\omega^* - \theta_0}{l_{t^*}} \rceil$ times. Hence $T^* = \{t^*, \ldots, t^*, \hat{t}\}$ with the size of T^* equals m and $\hat{t} \subseteq t^*$ is a feasible solution of Problem 1. Moreover, T^* is of miniminum size because $L_{T^*} = \omega^* - \theta_0$. Otherwise, if $T = \{t_1, \ldots, t_n\}$ is a feasible solution with $n < m$, then there would exist a t_i whose length is larger than the length of t^*, this contradicts the maximality of t^*. □

Corollary 2 *The MT-Problem can be solved in $O(n+m)$ time, where n is the number of steps in f and m is the size of a solution.*

Proof. The proof of Theorem 1 provides an additional insight on the optimal value when the SCA starts from a feasible configuration. If l_0 is the length of the interval in which the SCA meets the problem restrictions from the begining, then the minimum number of rotations of the SCA is $m = \lceil \frac{\omega^* - \theta_0 - l_0}{l_{t^*}} \rceil$. Finally, since the maximal interval t^* can be computed in linear time with a sweep from left to right, a greedy algorithm computes the optimal solution T^* in $O(n+m)$ time. □

4 Maximal energy collection

We say that a solar irradiance function f is unimodal if, for exactly one $i \in \{1, \ldots, n\}$, $\alpha_j \leq \alpha_{j+1}$ $\forall j < i$ and $\alpha_j \geq \alpha_{j+1}$ $\forall j \geq i$. Likewise, f is multimodal or k-modal if it has k local maxima. The unimodal case can be solved using a greedy approach. The main ideas are the following.

Given a real number l, let G_l be the maximum gain with respect to f of an interval of length l. By simplicity, we refer to G_l as the *maximum gain of length l*. Notice that when f is unimodal, any interval of maximum gain of length l contains intervals of maximum gain for lengths lower than l. Hence, given $l_1, l_2 \in \mathbb{R}$ with $l_1 \leq l_2 \leq \omega^*$, it can always be found t_1 and t_2 of lengths $l_{t_1} = l_1$ and $l_{t_2} = l_2$ of maximum gain in f for l_1, l_2, respectively, such that $t_1 \subseteq t_2$.

Theorem 3 *Let $l = \frac{\omega^*}{m}$ and t be a subinterval of $[0, \omega^*]$ s.t. $l_t = l$ and $g_t = G_l$. Then $T^* = \{t, ..., t\}$ with $|T^*| = m$ is optimal for MEC problem when f is unimodal.*

Corollary 4 *The MEC-Problem can be solved in $O(n+m)$ time when f is unimodal.*

When f is k-modal ($k > 1$), it is easy to find an example for which the MEC problem cannot be solved with the same greedy algorithm. Let us introduce some concepts to be used in the proposed solution. Recall that E is defined as the set of edges of f.

Definition 5 *An interval $t = (\theta_1, \theta_2)$ is discrete, called as a d-interval, if $\theta_1 \in E$ and $\theta_2 \in E$. The interval is semi-discrete if it starts or ends in an edge of f.*

Definition 6 *We say that a step of f is modal (m-step) if it is a local maximum.*

Definition 7 *An interval $t = (\theta_1, \theta_2)$ is an md-interval, if it is discrete and contains at least a modal step of f. A semi md-interval is a semi-discrete interval containing at least a modal step of f.*

The following results constitute the heart of our approach.

Lemma 8 *There exists an optimal solution T^* to the MEC problem s.t. for any $t \in T^*$, t is at least semi-discrete.*

Lemma 9 *There exists an optimal solution T^*, to the MEC problem s.t. $\forall i = 1 \ldots |T^*| - 1$, t_i is a discrete interval.*

Theorem 10 *There exist an optimal solution T^* to the MEC problem s.t. $\forall i = 1 \ldots |T^*| - 1$, t_i is an md-interval, and t_m is a semi md-interval.*

4.1 The algorithm

The following property of any optimal solution T^* can be easily proved: *removing any interval t_i from T^* yields a solution $T' = T^* - \{t_i\}$ that is optimal for $m-1$ moves and $\omega^* - l_{t_i}$ total displacement of the SCA*. This property, known as *optimal substructure property*, allows us to find an optimal solution by solving a collection of subproblems and it is the base of the greedy and dynamic programming paradigms. In addition, and more importantly, according to Theorem 10, the general form of the optimal solution to the MEC problem can be expressed as:

$$I^*_m = D^l_{m-1} + G_{\omega^*-l}, \tag{3}$$

being I^*_m the maximum gain associated to m moves, D^l_{m-1} the maximum gain for length at most l using $m-1$ discrete intervals, and G_{ω^*-l} the maximum gain in f for the remaining length. Because of the optimal substructure of the problem, D^l_{m-1} is optimal for length l. However, we cannot know beforehand the value of l, hence we divide the problem in two tasks:

Task 1: Computing D^l_{m-1}, $\forall l \in (0, \omega^*)$.

Task 2: Computing G_l, $\forall l \in (0, \omega^*)$.

According to (3), a solution with length l for the first task is associated to a solution with length $\omega^* - l'$ in the second, where $l' \leq l$ is the total length of the intervals obtained during the computation of D^l_{m-1}. In addition, notice that $l \in \mathbb{N}$ because the length of the steps of f are integers. Therefore, the following remarks can be stated:

Remark 1 *Combining the solutions from Task 1 and Task 2 takes $O(\omega^*)$.*

Remark 2 *I^*_m is the maximum value obtained after combining the solutions from Task 1 and Task 2.*

Task 2 can be easily solved in linear time for a given l and we have:

Theorem 11 *Task 2 can be solved in $O(n\omega^*)$ time.*

We now focus on solving Task 1. Since the considered intervals are discrete, we design an efficient algorithm based on Dynamic Programming (DP). Our algorithm will solve the MEC problem for any length considering only md-intervals, which is the requirement for Task 1. For simplicity, we refer to this version as the MEC-d problem.

Let B be the set containing the md-intervals of f. In addition, let us consider the table $D[i, j, l]$ indicating the maximum gain for the MEC-d problem when using up to interval i of B, with j movements and l as maximum solar displacement. Notice that intervals in B do not need to be sorted, but we assume a fixed order during the execution of the algorithm. Then, the update rule for D can be expressed as:

$$D[i,j,l] = \begin{cases} 0 & \text{(a) } 0 \in \{i,j,l\} \\ D[i-1,j,l] & \text{(b) } l < l_i \\ \max(D[i-1,j,l], & \text{(c) } \textit{else} \\ \quad g_i + D[i,j-1,l-l_i]) & \end{cases} \tag{4}$$

where g_i represents the gain of the interval i of B.

Theorem 12 *DP is optimal for the MEC-d problem and spends $O(n^2 m\omega^*)$ time.*

Remark 3 *For a given $l \in (0, \omega^*)$, $D[|B|, m-1, l]$ contains the optimal value for Task 1.*

The intervals corresponding to an optimal solution T^* to the MEC problem can be obtained after computing I^*_m. Notice that every decision is associated to an interval, both in Task 1 and Task 2; see Theorems 11 and 12. In Taks 2, the interval associated to G_l (for a given value of l) can be obtained by scanning f. On the other hand, for Task 1, it is easier to use the cases defining equation 4 to retrieve the intervals associated to a decision. Specifically, for any i, j, l, we check (a), (b) or (c); if cases (a) or (b) holds, then the candidate i is not used; if c holds, then we check the equality $D[i,j,l] = D[i-1,j,l]$ and if it holds, then candidate i is not used, otherwise, it is used. Starting this process at $D[|B|, m-1, l]$, being l the length of the optimal solution, it is possible to retrieve the full set of intervals in T^*.

Corollary 13 *The MEC problem can be solved in $O(n^2 m\omega^*)$.*

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