Some routing problems on a half-line with release times and deadlines

Alfredo García*[1] and Javier Tejel*[1]

[1]Departamento de Métodos Estadísticos and IUMA, Universidad de Zaragoza, Spain

The following problem is studied in [1, 3]. Let $N = \{1, 2, \ldots, n\}$ be a set of customers located on the real half-line $\mathbb{R}^+$ and let D be a depot located at $x = 0$. The distance (and also the travel time) from customer i to the depot is denoted by τ_i. A vehicle has to deliver goods from the depot to the customers. Each customer places an order to the depot and this order is associated with a time window $[r_i, l_i]$, with $l_i = r_i + S - \tau_i$. The release time r_i specifies the earliest possible time the vehicle can depart the depot to deliver at i. S can be seen as a service guarantee such that customer i cannot be served after $r_i + S$. Thus, $r_i + S - \tau_i$ is the latest dispatch time for customer i. For example, we can think of a restaurant delivering meals at home as a depot, the release time of an order (a customer ordering for a meal) as the time that the order can be dispatched from the restaurant after preparing the meal, and S as the time in which the restaurant guaranties that the order will be delivered.

The problem analyzed in [3] is determining the minimum possible completion time c^* of a schedule of delivery routes that can be executed by a single driver, each starting and ending at the depot, such that each order i is dispatched at or after r_i and delivered at or before $r_i + S$. Any feasible solution to the problem will consist of a set of k routes, visiting a subset of customers in each route. Assuming that the customers are ordered according to their release times, that is, $r_1 \leq \ldots \leq r_n$, it is proved in [3] that there is always an optimal delivery schedule with non-interlacing routes, where two routes K_1 and K_2, with $\min\{i|i \in K_1\} < \min\{j|j \in K_2\}$, are non-interlacing if and only if $\max\{r_i|i \in K_1\} < \min\{r_j|j \in K_2\}$. As a consequence, in any delivery schedule with non-interlacing routes, the customers visited in any route have consecutive release times.

Geometrically, one can imagine the set of customers as a set of n horizontal segments $s_1, \ldots, s_n$ such that the y-coordinate of s_i is τ_i and the x-coordinates of the endpoints of s_i are r_i and l_i, respectively. The abscissa axis represents time. For instance, Figure 1 shows a set of eight customers and a feasible schedule (in red), consisting of three routes, K_1, K_2 and K_3, to serve the eight customers. After waiting at the depot, K_1 starts at r_2, dispatches orders 2, 1 and 3, and ends at $r_2 + 2\tau_3$. K_2 starts at $r_2 + 2\tau_3$, dispatches orders 4 and 5, and ends at $r_2 + 2\tau_3 + 2\tau_5$. Finally, after waiting again at the depot for a while, K_3 starts at r_8, dispatches orders 8, 7 and 6, and ends at $r_8 + 2\tau_6$.

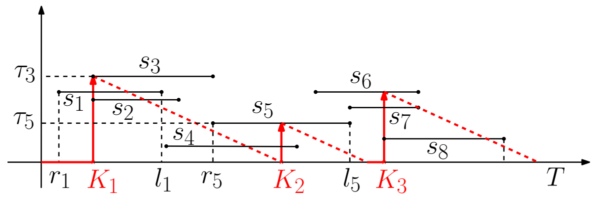

Figure 1: A feasible schedule to serve eight customers.

Let $c(i)$ be the minimum completion time of a non-interlacing schedule serving orders $\{1, \ldots, i\}$, or ∞ if it is not possible to serve $\{1, \ldots, i\}$ feasibly with a single server. Thus, c^* will be given by $c(n)$. Defining $c(0) = 0$, the following recurrence [3] allows one to compute $c(i)$, for $i = 1, \ldots, n$:

$$c(i) = \min_{0 \leq j < i} \left\{ \max\{c(j), r_i\} + 2 \max_{j<k\leq i}\{\tau_k\} \mid \max\{c(j), r_i\} \leq \min_{j<k\leq i}\{l_k\} \right\}$$

In this talk, we will show how to solve this recurrence in $O(n \log n)$ time, improving the $O(n^2)$ algorithm given in [3]. In addition, using the algorithm described in [2], if $S = \infty$, that is, there are no deadlines, then the previous recurrence can be solved in $O(n)$ time, improving the $O(n^2)$ algorithm provided in [3].

References

[1] C. Archetti, D. Feillet and M.G. Speranza, Complexity of routing problems with release times, *European Journal of Operational Research* **247** (2015), 797–803.

[2] L. Larmore and B. Schieber, On-line dynamic programming with applications to the prediction of RNA secondary structure, *J. of Algorithms* **12** (1991), 490–515.

[3] D. Reyes, A.L. Erera and M.W.P. Savelsbergh, Complexity of routing problems with release dates and deadlines, *European Journal of Operational Research* **266** (2018), 29–34.

*Emails: {olaverri, jtejel}@unizar.es. Research supported by project PID2019-104129GB-I00 / AEI / 10.13039/501100011033 of the Spanish Ministry of Science and Innovation, and by Gobierno de Aragón project E41-23R.